Vagueness, Open texture and Computational dialectics

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Abstract:

In this paper we are seeking general definitions of vagueness. They have much to do with dialectical reasoning since we are interested in vagueness caused by differing opinions, or points of view, about the definition of concepts. After giving three definitions for vagueness we sketch ways of choosing meaning for vague concepts, using "direct democracy". In this context we also define open texture and a related concept, porosity.

I am currently working on my Ph.D. thesis at the University of Montreal under the direction of Paul Bratley, of the *Département d'informatique et de recherche opérationnelle*, and Daniel Poulin of the *Centre de recherche en droit public*. Our team's main subject of research is legal interpretation with differing points of view to reproduce the adversarial dimension in legal reasoning (see [Bratley 94] and [Poulin 93]). This paper is an offspring of that work.

1) Vagueness and Open texture

Although vagueness is a vague concept, this section contains a few "natural language" definitions intended to shed some light on the subject.

Quoting Susskind ([Susskind 87], p. 187), vagueness can be seen as a form of apparent imprecision:

> [...] words are vague when they clearly have no definite set of necessary and sufficient conditions governing their use and application. Terms such as 'fair' and 'reasonable', in this sense, can be seen as vague.

We can say that concepts are vague because their meaning is varying along time, points of view, application domains, etc. The variations make impossible to cast single definitions with fixed necessary and sufficient conditions. Open texture, then, is a possibility of vagueness (id., quoting Waismann):

> *Vagueness* should be distinguished from *open texture*. A word which is actually used in a fluctuating way (such as ‘heap’ or ‘pink’) is said to be vague; a term like ‘gold’, though its actual use may not be vague, is non-exhaustive or of an open texture in that we can never fill up all the possible gaps through which a doubt may seep in. Open texture, then, is something like *possibility of vagueness*.

In the legal field the presence of open texture usually implies that even when the intended meaning looks clear, i.e. the terms are not used in vague way, there are possibilities of debate in some situations, whether hypothetical or real.

Here, we will also use the term ‘Porosity’, from the German expression coined by the German philosopher Waismann, “Porösität der Begriffe” (literally “Porosity of the concept”, see [Gordon 93], page 26), to name the fact that some concepts can’t be defined once and for all. Somehow ‘porosity’ implies the necessity of vagueness, in the modal logic sense. Hence the term ‘porosity’, which gives the idea that the extension of these concepts is like unstable lather. We will clarify this idea in the fourth section.

Although it may seem questionable, we will explore vagueness through contention. Thus we consider a term to be vague whenever there is some debate about its exact meaning. The motivation behind this stance is twofold. First, dialectical reasoning is precisely a means to uncover, through debate, the possible imprecision of terms. If meanings were always established in a straightforward, simple and deterministic manner there could hardly be such things as vagueness and open texture. Furthermore, we claim that not only is dialectical reasoning A means to explore vagueness but that it is, in fact, the ONLY means to do so. While it may be said that this assertion is both unprovable and irrefutable, it seems clear that if we are to study vague terms we must look at the multitude of meanings they have, and therefore must use a form of dialectical reasoning. This can be direct confrontation between people, hypothetical reasoning within one’s mind, study of historical variations of meanings, etc.

This will surely be matter of discussion during the workshop.

2) The Dialectical space

In this section we give simple definitions for the concept of vagueness, considering only conclusions without arguments. To achieve this in a dialectical context we use set theoretic combinations of extensions of predicates for different agents. The dialectical reasoning considered here is not classical. It is N-way instead of two-way and it is not restricted to opposition but also includes variations amongst opinions.

First, assume we have agents defined this way:

$\alpha_i = \langle \vdash_i, K_i \rangle$, where $|\!-_i$ is some mechanism by which α_i can derive conclusions from its knowledge in K_i. These agents are very general, indeed an agent can be almost anything under this definition. This is possible because we are only interested in the conclusions of the agents, not in their cognitive processes.

Then define an agent sequence S to be a list of agents:

$S = [\alpha_1, \alpha_2, ... \alpha_n]$

These sequences can represent many phenomena, for instance:

- A society of persons, even the whole of mankind, with any arbitrary ordering, if we suppose that this representation of people as agents with a single derivability relation and a single knowledge base is possible[1].
- Different states of a single mind. Then we might have something akin to default reasoning, that is knowledge present in some K_i may be retracted in a later K_j.
- If the even α_i represent the states of mind of one actor at each turn of a two player game, and the odd α_j represent the states of another, then the sequence is like Gordon's Pleadings Game [Gordon 93].
- Different experts, each having his own point of view.
- The Ks might contain different case bases, and then we would be closer to case based reasoning, as in Ashley's HYPO [Ashley 90].

Here we need not be concerned with the relationships between the agents, but we assume they can exchange information if necessary. We also assume their conclusions are formulated symbolically.

In the remainder of this text, we use the notation $\alpha_i \varnothing \phi$ to mean that the agent α_i can conclude a variable free atomic sentence ϕ, that is $K_i |\!-_i \phi$ where ϕ is atomic and variable free. In other terms :

For any agent α

If $\alpha \varnothing p(x_1..x_m)$ Or $\alpha \varnothing \neg p(x_1..x_m)$ Then $x_1..x_m$ are terms without variables

In the previous definition series are not 'instantiated', they are not aimed at a "subject of discussion". There is not necessarily any common knowledge between the agents in agent sequences.

To instantiate the sequences we have to assume that all the agents are at least talking about the same thing. To achieve this, the domain of the interesting predicates must be the same for all the agents, at least implicitly. That is, for an interesting predicate symbol 'p', we have:

Same_domain(p, $[\alpha_1..\alpha_n]$) Iff

For all agents α, β in $[\alpha_1..\alpha_n]$

If $\alpha \varnothing p(x_1..x_m)$ Or $\alpha \varnothing \neg p(x_1..x_m)$ And

$\beta \varnothing p(y_1..y_m)$ Or $\beta \varnothing \neg p(y_1..y_m)$ Then

the domains of x_i and y_i are the same for all i in [1..m]

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[1] Because we are not limiting the derivability relation nor the knowledge base to purely symbolic forms, this should be acceptable even to the 'Turing test' adversaries.

This simply means that the domains of the intended models of ‘p’ are the same for all agents. If the agents are identically designed, e.g. expert systems all built by the same team, this is not a problem. However if the agents are either heterogeneous in origin and design or if they are humans, intelligent creatures with varying pasts, this property might not be possible to verify in practice. Then we assume that the agents can exchange messages, in a manner we will not be concerned with here, to ensure that the domain of each interesting predicate is the same for all. The system of education of a given society is, amongst others things, such a system to teach a common language by way of exchanged messages.

Then we define dialectical sequences:
an agent sequence ‘as’ is a dialectical sequence for the predicate symbols pset={$p_1..p_k$} iff

- For all p in pset : Same_domain(p, as)
- For all α in as : $\alpha \varnothing p(x_1..x_n)$ or $\alpha \varnothing \neg p(x_1..x_n)$ for some $x_1..x_n$

As can be seen, the definition implies that sequences with agents having “no opinion” about a predicate are not accepted as dialectical sequences. Therefore we will consider sequences where the agents are knowledgeable about the points, at least partially. We do not enforce differing opinions amongst the agents, although the interesting cases occur when they differ. Finally we will generally assume that there is some positive knowledge (i.e. there exists α such that $\alpha \varnothing p(x_1..x_n)$).

Now, starting with the following auxiliary definitions, where ‘p’ is a predicate symbol:

Positive(p, α) = { $p(x_1..x_n) \mid \alpha \varnothing p(x_1..x_n)$ }	The positive extension of ‘p’ for agent ‘α’
Negative(p, α) = { $p(x_1..x_n) \mid \alpha \varnothing \neg p(x_1..x_n)$ }	The negative extension of ‘p’ for agent ‘α’

We define these functions combining extensions of the predicate ‘p’ for all agents in the dialectical sequence ‘ds’, with pset={p}:

$Maxp(p, ds) = \bigcup_{a \in ds} Positive(p, a)$	The total positive extension of 'p'
$Minp(p, ds) = \bigcap_{a \in ds} Positive(p, a)$	The common positive extension of 'p'
$Maxn(p, ds) = \bigcup_{a \in ds} Negative(p, a)$	The total negative extension of 'p'
$Minn(p, ds) = \bigcap_{a \in ds} Negative(p, a)$	The common negative extension of 'p'
$Mink(p, ds) = \bigcap_{a \in ds} (Positive(p, a) \approx Negative(p, a))$	The common knowledge of 'p'
$Max(p, ds) = Maxp(p, ds) \setminus Maxn(p, ds)$	The part of Maxp not contradicted in Maxn
$Min(p, ds) = Minp(p, ds) \setminus Minn(p, ds)$	The part of Minp not contradicted in Minn

We now give three tentative definitions for the concept of vagueness within this general framework.

The first definition is very fussy about vagueness. Here, a concept, named by 'p', is considered to be vague whenever there is disagreement about its meaning in a dialectical sequence 'ds', assuming the interesting case where there is some positive knowledge :

$$\begin{aligned} Vague1(p, ds) \Longleftrightarrow\ & (\ Minp(p, ds) \neq Maxp(p, ds) \text{ Or} \\ & \ \ Minn(p, ds) \neq Maxn(p, ds)\ ) \\ & \text{And } Maxp(p, ds) \neq \emptyset \end{aligned}$$

The problem with this definition is that a predicate is considered Vague1 as soon as there is a slight disagreement about its extension. Thus it could easily be that one of the agents is wrong or simply that they have a differing amount of knowledge about the predicate.

Second, we consider a very narrow version of vagueness:

$$\begin{aligned} Vague2(p, ds) \Longleftrightarrow\ & Min(p, ds) = \emptyset \\ & \text{And } Maxp(p, ds) \neq \emptyset \end{aligned}$$

This definition suggests that a concept is vague if there is no unanimous core of agreement about its content, excluding the trivial case where no agent has positive knowledge about it. It is comparable to the point of view of Susskind about vagueness: when there is no accepted necessary and sufficient conditions defining the concept.

This third and last version is an intermediate between Vague1 and Vague2:

$$\begin{aligned} Vague3(p, ds) \Longleftrightarrow\ & Mink(p, ds) \setminus (Minp(p, ds) \approx Minn(p, ds)) \neq \emptyset \\ & \text{And } Maxp(p, ds) \neq \emptyset \end{aligned}$$

Here a concept is considered vague when there are agents contradicting parts of the core of knowledge, whether positive or negative, about ‘p’. In this case, all the agents have knowledge for some values “$x_1..x_m$” in “p($x_1..x_m$)”, but some agents say “p($x_1..x_m$)” and others “¬ p($x_1..x_m$)”. So we have Vague3 if there is some contradiction among the agents, but this contradiction is not necessarily total as is the case with Vague2.

<u>Example</u>:

Three agents (A1, A2 and A3) have differing opinions about what objects are vehicles (pset={Vehicle}). We have, in the second row of this table, Positive ≈ Negative for each agent:

agent A1	agent A2	agent A3
Vehicle(Car)	Vehicle(Car), Vehicle(Truck)	Vehicle(Car), Vehicle(Truck), Vehicle(Bicycle)

Hence, Maxp(Vehicle, [A1, A2, A3]) = { Vehicle(Car), Vehicle(Truck), Vehicle(Bicycle) } and Minp(Vehicle, [A1, A2, A3]) = { Vehicle(Car) }

Therefore according to Vague1, Vehicle is a vague concept. This is because Vehicle has varying extensions. We have neither Vague2 nor Vague3 because no agent contradicts the others’ knowledge and there is core knowledge, that is Vehicle(Car).

<u>Example</u>:

Now, let us see how Vague2 can be verified. Again, the following table gives the opinions for the predicate Vehicle:

agent A1	agent A2	agent A3
Vehicle(Bicycle)	Vehicle(Bicycle), Vehicle(Truck)	¬ Vehicle(Bicycle), Vehicle(Truck), Vehicle(Car)

We see there is no core of agreement around what are vehicles, so Min(Vehicle, [A1, A2,A3]) = ∅. Indeed we even have Minp(Vehicle, [A1, A2,A3]) = ∅. The contrary, Min = ∅ and Minp ≠ ∅, would have meant a contradiction from one agent around the core of knowledge.

<u>Example</u>:

This example illustrates Vague3:

agent A1	agent A2	agent A3
Vehicle(Car), Vehicle(Truck)	Vehicle(Car), Vehicle(Truck)	Vehicle(Car), ¬ Vehicle(Truck), Vehicle(Bicycle)

We simultaneously have Vague1, as there is some difference, and Vague3 because there is a contradiction between the total knowledge of the agents (Maxk) and the knowledge of agent A3: ¬ Vehicle(Truck). But we do not have Vague2 as there is a “core of agreement” around Vehicle(Car).

3) Choosing a definition

The presence of vague concepts does not mean we have to remain passive before a problem. In this section we will describe two methods of choosing a suitable definition for vague concepts. Since the definitions can evolve with time we will suppose that the choice is made at a precise moment in time. These methods, called choice functions, will be used in the next section to define open texture and porosity.

Here the choice functions have recourse to what we might call “direct democracy”. The choice of a definition for a vague concept is given to a majority of agents. Of course, any other kind of choice function could be defined, for instance a “tyrannical” function would consider the opinion of only one agent.

First the direct democracy choice function Unanimity is defined this way, where ‘p’ is a predicate and ‘ds’ a dialectical sequence for pset={p}:

Unanimity(p, ds) = Min(p, ds)

Hence, Unanimity chooses the definitions corresponding to the minimal core of knowledge around ‘p’. Assuming Maxp(p, ds) ≠ □ always holds in the interesting cases, there is unanimity if and only if Vague2 does not hold.

<u>Example</u>:

Following the situation given in the first example of the previous section:

agent A1	agent A2	agent A3
Vehicle(Car)	Vehicle(Car), Vehicle(Truck)	Vehicle(Car), Vehicle(Truck), Vehicle(Bicycle)

We have Unanimity(Vehicle, [A1, A2, A3]) = { Vehicle(Car) }

While Unanimity is very restrictive, the following choice function implements the usual “majority by number”:

Majority(p, ds) = { p(x1..xn) | p(x1..xn) □ Maxp(p, ds) And
Card({a in ds | a ∅ p(x_1..x_n)})
> Card({a in ds | a ∅ ¬ p(x_1..x_n)}) }

The choice function Majority thus chooses a definition of ‘p’ using a count of the differing opinions. The positive opinions are accepted if they are not countered by a greater number of negative ones. This version of majority does not always enforce a choice: if no positive definition is chosen by majority then Majority remains empty.

Here the decision is taken for the positive side only. This is much like in parliaments where laws are accepted if there are majorities for them, but where negations of laws are not enforced in case of rejection. One can imagine other definitions of majority, each adapted to some end.

Example:

Using the same example, we have:

Majority(Vehicle, [A1, A2, A3]) = { Vehicle(Car), Vehicle(Truck), Vehicle(Bicycle) }

And we can see that Vehicle(Bicycle) is accepted since neither A1 nor A2 explicitly rejects it.

Example:

If there is some kind of closed world assumption in the agents, the example becomes:

agent Cw1	agent Cw2	agent Cw3
Vehicle(Car), ¬ Vehicle(Truck), ¬ Vehicle(Bicycle)	Vehicle(Car), Vehicle(Truck), ¬ Vehicle(Bicycle)	Vehicle(Car), Vehicle(Truck), Vehicle(Bicycle)

And then:

Majority(Vehicle, [Cw1, Cw2, Cw3]) = { Vehicle(Car), Vehicle(Truck) }

4) Open texture and Porosity

Now we are ready to give definitions for open texture and porosity. Many people can have their own perception of open texture and porosity and their meaning is subject to debate. The definitions given here take a "political evolution" perspective as they are based on choice functions. They are also based on the idea that open texture and porosity are relative to:

- some dialectical sequence, presumably evolving with time,
- a sequence of time points,
- a choice function,
- a vagueness definition (e.g. Vague1, Vague2 or Vague3).

Assuming that the agents 'ds' are immersed in time we can study the evolution of their opinions about the predicate 'p'. We construct the following dialectical sequence:

Let 'ds' be a dialectical sequence $[\alpha_1..\alpha_n]$

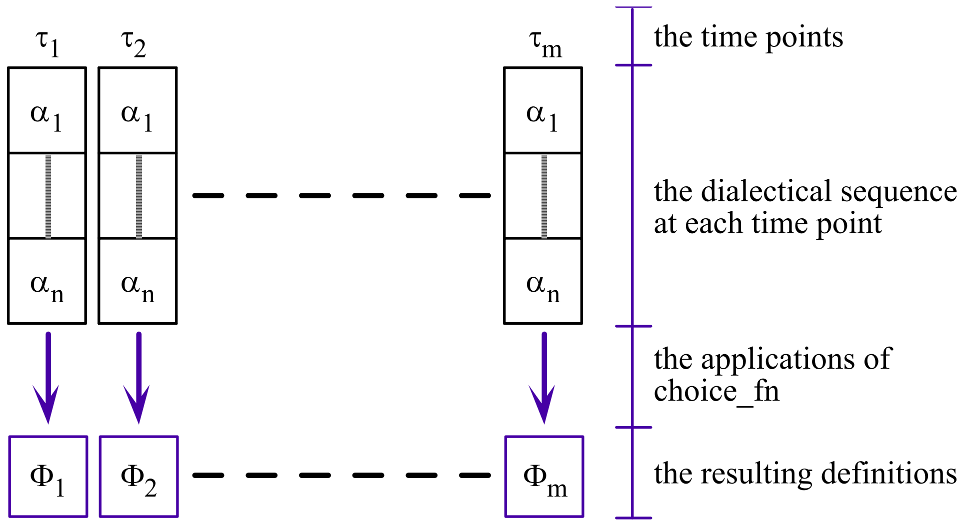
Let 'tp' be a sequence of time points $[\tau_1..\tau_m]$, then

Opinions(p, ds, tp, choice_fn) is a dialectical sequence where in each agent $\alpha_i = <|\!-_i, K_i>$ we have

K_i = the 'ds' at time τ_i

$|\!-_i$ = the choice function applied to 'p' and K_i, i.e. choice_fn(p, K_i)

This figure illustrates the construction, with arrows showing the application of 'choice_fn' (i.e. $|\!-_i$):



'Opinions' thus represents the variation of the opinions of a dialectical sequence along time. This is like sampling the dialectical sequence $[\alpha_1..\alpha_n]$ at the time points $[\tau_1..\tau_m]$, and filtering the results with some choice function such as Unanimity, Majority or any other. The Φ_i are the results of the filtering.

Example:

Let p=Vehicle, ds={A1, A2, A3}, tp={T1, T2, T3} and choice_fn = Majority

In the last row Φ_i is the result of 'choice_fn' on 'ds' at each T_i

	T1	T2	T3
A1	Vehicle(Car)	Vehicle(Car), Vehicle(Truck)	Vehicle(Car), Vehicle(Truck)
A2	Vehicle(Car)	Vehicle(Car), Vehicle(Truck)	Vehicle(Car), Vehicle(Truck), Vehicle(Bicycle)
A3	¬ Vehicle(Car)	Vehicle(Car)	Vehicle(Car), ¬ Vehicle(Bicycle)
Φ_i	Vehicle(Car)	Vehicle(Car), Vehicle(Truck)	Vehicle(Car), Vehicle(Truck)

Here there is Majority for Vehicle(Car) at all time points and for Vehicle(Truck) starting from T2. The inclusion of Vehicle(Bicycle) by agent A2 at time T3 is countered by the rejection of A3.

Finally, the definitions for porosity and open texture both depend on the opinions as computed by some Opinion function and on a definition for the concept of vagueness.

Assuming a fixed Opinions(p, ds, tp, choice_fn) for some fixed 'ds', 'tp' and 'choice_fn', and assuming 'vague_fn' is a definition for vagueness, then:

a) Open_textured(vague_fn) <=> vague_fn(p, sds) for some subsequence sds ∏ Opinions(p, ds, tp, choice_fn)

b) Porous(vague_fn) <=> vague_fn(p, sds) for all subsequences sds (with more than one element) such that sds ∏ Opinions(p, ds, tp, choice_fn)

Thus open texture is “possibility of vagueness” and porosity is “necessity of vagueness” in the sociopolitical evolution of the opinions about ‘p’, like in the informal definitions given in the first section.

Example:

In the context of Opinions(Vehicle, [A1, A2, A3], [T2, T3], Majority) of the last example we do not have Porous(Vague1) since Vague1 is false for the subsequence Opinions(Vehicle, [A1, A2, A3], [T2, T3], Majority). On the contrary we have Open_textured(Vague1) because the subsequence Opinions(Vehicle, [A1, A2, A3], [T2, T3], Majority) is Vague1. Using Vague2 we neither have Porous nor Open_textured.

5) Conclusion

In this paper we have presented definitions of vagueness using dialectical sequences of agents. The sequences are general and can represent many phenomena. The definitions of vagueness rely on set theoretic properties of the conclusions of agents. Then we have shown two simple methods of choosing definitions for vague concepts from the opinions of the agents. Finally we gave definitions for open texture and porosity taking the perspective of social evolution.

We believe that we have shown, to a reasonable account, that vagueness, open texture and porosity are intimately related to dialectics.

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